NBA Chemistry: Positive and Negative Synergies in Basketball

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Abstract

We introduce a novel Skills Plus Minus ("SPM") framework to measure on-court chemistry in professional basketball. First, we evaluate each player's offense and defense in the SPM framework for three basic skill categories: scoring, rebounding, and ball-handling. Next, we simulate games using the skill ratings of the ten players on the court. Finally, we calculate the synergies of each NBA team by comparing their 5-player lineup's effectiveness to the "sum-of-the-parts." We find that these synergies can be large and meaningful. Because skills have different synergies with other skills, our framework predicts that a player's value depends on the other nine players on the court. Therefore, the desirability of a free agent depends on the current roster. Indeed, our framework generates mutually beneficial trades between teams. Other ratings systems cannot generate ex-ante mutually beneficial trades since one player is always rated above another. We find more than two hundred mutually beneficial trades between NBA teams, situations where the skills of the traded players fit better on their trading partner's team. We also find that differences in synergies between teams explain as much as six wins and that teams are no more likely to exhibit positive chemistry than negative chemistry.

KEYWORDS: NBA, SYNERGY, CHEMISTRY, SKILLS PLUS-MINUS

Introduction

> "My model for business is The Beatles. They were four guys who kept each other's negative tendencies in check. And the total was greater than the sum of the parts. Great things in business are not done by one person; they are done by a team of people."
>
> – Steve Jobs

Basketball, one of the world's most popular and widely viewed sports, is a timed game played by two teams of five players on a rectangular court1. While the exact playing regulations vary across different governing bodies, we focus on the National Basketball Association ("NBA"), which is widely considered the premier men's professional basketball league in the world. Teams alternate possession of the basketball and attempt to score points by shooting a ball through a hoop 18 inches in diameter and 10 feet high mounted to a backboard at each end of the floor. The team with the most points at the end of the game wins the game. In the NBA,

[1] Background information on the game of basketball draws from http://en.wikipedia.org/wiki/Basketball.

teams have 24 seconds to attempt a field goal. A successful field goal attempt is worth two points for the shooting team, or three points if the shooting player is behind the three-point line. A free throw is awarded to an offensive player if he is fouled while shooting the ball. A successful free throw attempt is worth one point. Each possession ends with either a field goal attempt, free throw attempt, or a turnover (if a player loses possession to the opposing team). Turnovers can occur when the ball is stolen (a “steal”) or if the player steps out of bounds or commits a violation (“non-steal turnover”). A missed field goal attempt or free throw attempt results in a rebounding opportunity, where the teams fight to gain possession of the ball. Each possession ends with a finite number of possible outcomes, making the simulation of a game feasible.

The rules of basketball do not specify any positions whatsoever, and there are no special positions such as goalie. Over time, positions have evolved, where shorter and quicker players play “guard”, a position that requires more ballhandling, passing and outside shooting. Meanwhile, taller and stronger players typically play “forward” or “center”, operate closer to the basket, and grab more rebounds. Traditionally, teams play with two guards, two forwards, and one center, but it is possible to play with five guards or five centers, if a team so desires.

A box score summarizes the statistics of a game, detailing player contributions such as minutes played, field goal attempts, successful field goals, free throw attempts, successful free throws, rebounds, assists, steals, blocks and turnovers. Assists are awarded when a player passes the ball to a teammate who then scores a field goal. A block occurs when a defensive player legally deflects a field goal atatempt by an offensive player. In general, guards accumulate more assists, while centers block more shots. There have been many attempts to rate individual basketball players using box score statistics. Examples include Wins Produced or Win Shares (see Oliver 2004). These ratings systems generally agree with expert opinions on the best players in the league. For example, during the 2012-2013 season, both Wins Produced and Win Shares suggested that LeBron James and Kevin Durant were the two best players in the NBA. These two players also finished first and second in Most Valuable Player (“MVP”) voting for that season.

While these box score ratings can measure an individual’s contributions, they do not necessarily explain how players interact on the court. For example, it is possible that the five best players in the NBA are all centers. In this case, a team with five centers may not be the optimal lineup, since there would be no one to bring the ball up the court or guard the quicker opposing guards. Therefore to determine the optimal lineup, we would want to measure the “synergies” among players, and predict which players play well with each other. Our paper attempts to address this issue by introducing a Skills Plus Minus (“SPM”) framework that decomposes a player’s contributions into three skills: scoring, rebounding and ball-handling.

In sports, synergies are not often applied to individual athletes. Bollinger and Hotchkiss (2003) in evaluating baseball define team synergy as firm-specific productivity such as the signals and strategies unique to the team. MacDonald and Reynolds (1994) explicitly avoid attention to “synergy” or “chemistry” and focus only on the value of each baseball player on his own. Indeed, they hypothesize “a reasonably efficient market in player talent and a consequently quasi-efficient assignment of players among teams and within team line-ups.” Idson and Kahane (2000) begin the path of testing this hypothesis by separating out the effects of individual and team productivity on salary determination in the National Hockey League, and indeed find that team attributes not only directly affect individual pay, but can also diminish certain individual productivity effects. Their results in fact hint at synergies: they find complementarity across some productive attributes but not others and they hypothesize that “larger, more significantly positive interactions might follow if certain positions are paired.”

They leave this open as a fruitful subject of future research.

Here we are able to actually test this hypothesis by using a large dataset of repeated interactions combined with our Skills Plus Minus model and framework to decompose the players into their constituent skill groups and evaluate the synergies resulting from various combinations of those skill groups. We find that the allocation of players within teams is not efficient, and that there are hundreds of trades that would have benefitted both trading teams because of the effects on team chemistry.

An example helps frame our argument. With the third pick in the 2005 NBA draft, the Utah Jazz selected Deron Williams, a 6’3” point guard who played collegiately at Illinois. Using the very next pick, the New Orleans Hornets drafted Chris Paul, a 6’0” point guard from Wake Forest. Since the moment they entered the league, the careers of Williams and Paul have often been compared. Countless debates and discussions sparked about who is the better point guard. There are arguments for both sides.

The box score statistics seem to favor Paul. His career statistics (18.7 points per game, 4.6 rebounds, 9.9 assists, 2.4 steals, 0.571 true shooting percentage (“TS%”)) are better than Williams across the board (17.2 points, 3.2 rebounds, 9.2 assists, 1.1 steals, 0.560 TS%). Paul has played in more All-Star games (4 vs. 2) and appeared on more All-NBA teams (3 vs. 2).

Meanwhile, supporters of Williams point to his better regular season record (0.590 winning percentage vs. 0.555 for Paul), relative playoff success (20 playoff wins vs. 10), head-to-head record against Paul, size, strength, and durability. They argue that Williams is a stronger one-on-one defender who does not gamble for steals.

At the end of the 2009-2010 season, if Utah had traded Deron Williams for Chris Paul, would they have been better off? If New Orleans had traded Chris Paul for Deron Williams, would they have been better off? Using the framework introduced in this paper, we can answer these questions: surprisingly, the answer is YES to both. A Williams-for-Paul swap would have made both teams better off and is an example of a mutually beneficial trade. Such a trade should not have been possible if team composition were efficient; at the very least, such a trade should have been consummated, but it never was.

This paper introduces a novel Skills Plus Minus framework to measure on-court chemistry in basketball. This SPM framework builds upon the Advanced Plus Minus (“APM”) framework first introduced by Rosenbaum (2004). While APM evaluates each player based on the points scored while they are in the game, SPM evaluates each player based on the offensive and defensive components of three basic categories of skills: scoring, rebounding and ball-handling. For example, a player’s “steal” ratings (part of the ball-handling category) are determined by how many steals occur while he is in the game. Like APM, SPM considers the other nine players on the court. A benefit of the APM and SPM framework is the ability to capture skills that are not found in traditional box score measures, such as off-the-ball defense, boxing out, and setting picks. Also, in contrast to other ratings such as Wins Produced, APM and SPM do not make position and team adjustments to the player ratings.

We use the SPM framework to simulate games using the skill ratings of the ten players on the court. These simulations incorporate how each play starts: out-of-bounds, steal, defensive rebound or offensive rebound. We find these starting conditions materially affect the outcome of the possession. The simulations are then used to measure the effectiveness of individual players and 5-player lineups.

We investigate which basketball skills have synergies with each other. Traditionally, team chemistry has been difficult to measure. Berri and Jewell (2004) use roster stability as a proxy for chemistry. While they acknowledge the "potential impact of disruptive players," (which we would call negative synergies in our framework) they note that "identifying and quantifying the impact of such players appears problematic." Our framework solves this problem.

Another method to measure chemistry compares the "lineup APM" versus the sum of the constituent single player APM's. The problem with that approach is that there are too many possible five-player lineup combinations. The APM's of the five-player lineups have small sample problems since the minutes played of any given five-player lineup can be small. Our innovation is that we are able to predict synergies while avoiding this problem.

We calculate the synergies of each NBA team by comparing their 5-player lineup's effectiveness to the "sum-of-the-parts." These synergies can be large and meaningful. Because skills have different synergies with other skills, a player's value depends on the other nine players on the court. Therefore the desirability of a free agent depends on the players currently on the roster.

Finally, our framework is able to generate mutually beneficial trades. Other ratings systems cannot generate mutually beneficial trades, since one player is always rated above another, c.f. Kubatko, Oliver, Pelton, and Rosenbaum (2007) for a review of most of them, or Berri (1999) or Berri (2008) for more detail on Wins Produced. Berri and Brook (1999) investigate whether trades are ex-post mutually beneficial and argue that trades can be ex-ante mutually beneficial if the ex-post distribution of minutes is known and different. In contrast, our framework generates ex-ante mutually beneficial trades without a change in the distribution of minutes played. Using our framework, we find many mutually beneficial trades, when the skills of the traded players fit better on their trading partner's team. One such mutually beneficial trade is Chris Paul for Deron Williams.

Methods

Description of the Data

While our primary innovation is a theoretical framework to model on-court chemistry, we use data to illustrate. Berri and Schmidt (2010) criticize APM because the player ratings are not stable from year-to-year. They favor ratings that use box score statistics (e.g. Wins Produced), because the ratings are more predictable from year-to-year. We acknowledge Berri and Schmidt's criticism and therefore use data from four NBA seasons (2006-2007 through 2009-2010) to achieve better estimates for player skills. While Fearnhead and Taylor (2011) allow their APM ratings to be time-varying, we estimate one rating for all four years.

The data we use is from basketballgeek.com, maintained by Ryan J. Parker, and represents a processed version of the play-by-play information from the NBA and ESPN. The data includes the names of all players on the court at each time, the location of the shots taken, result of possession, and more. The data set includes 4,718 games and 987,343 plays.

Tables 1-4 display summary statistics from our data set.

Table 1. Possession Start Variables

Possession Start	Count	Percent
Defensive Rebound	256,589	26.0%
Offensive Rebound	104,903	10.6%
Steal	59,329	6.0%
Out of Bounds	566,522	57.4%
Total	987,343	100.0%

Table 2. Possession Outcomes

Possession Outcomes	Count	Percent
Steal	68,460	6.9%
Non-steal turnover	66,912	6.8%
Missed FT – 2 pts	5,953	0.6%
Missed FT – 1 pt	15,068	1.5%
Missed FT – 0 pts	7,161	0.7%
Made FT – 3 pts	16,650	1.7%
Made FT – 2 pts	59,746	6.1%
Made FT – 1 pt	19,908	2.0%
Missed 3 FG	108,651	11.0%
Made 3 FG	60,652	6.2%
Missed 2FG	298,416	30.3%
Made 2 FG	257,524	26.1%
Total	985,101	100.0%

Table 3. Offensive Rebounds

Type	OReb	Missed Shots	OReb%
Field Goal	127,489	407,154	31.3%
Free Throw	3,749	28,218	13.3%

Table 4. Players Involved in the Most Plays in Our Data Set.

	Name	Plays	Name	Plays	Name	Plays	Name	Plays	Name	Plays	Name	Plays
1	Andre Iguodala	53,798	Samuel Dalembert	39,505	Ronnie Brewer	29,750	Brook Lopez	22,937	Mike James	17,691	Kris Humphries	12,515
2	Kobe Bryant	50,783	LaMarcus Aldridge	39,388	Antonio McDyess	29,712	Jason Maxiell	22,841	Michael Beasley	17,603	Josh Powell	12,290
3	Dwight Howard	49,297	Zach Randolph	39,098	Zydrunas Ilgauskas	29,589	Aaron Brooks	22,826	Eddy Curry	17,458	Leon Powe	12,131
4	LeBron James	49,254	Carlos Boozer	38,806	Luke Ridnour	29,559	Carlos Delfino	22,801	Jamaal Tinsley	17,428	Renaldo Balkman	12,010
5	Antawn Jamison	48,399	Allen Iverson	38,764	T.J. Ford	29,412	Jason Williams	22,734	C.J. Miles	17,399	Tony Battie	11,894
6	Jason Kidd	47,746	Mike Miller	38,742	Luis Scola	29,352	Jordan Farmar	22,712	Marko Jaric	17,225	Tyreke Evans	11,793
7	Andre Miller	47,515	Mike Bibby	38,565	Peja Stojakovic	29,175	Linas Kleiza	22,674	Josh Childress	17,156	Jamaal Magloire	11,681
8	Rudy Gay	47,238	Kevin Durant	38,436	DeShawn Stevenson	29,124	Daniel Gibson	22,402	Wally Szczerbiak	17,126	Ersan Ilyasova	11,617
9	Joe Johnson	47,209	Kirk Hinrich	38,322	Andres Nocioni	28,806	Dahntay Jones	22,334	Fabricio Oberto	17,051	Brent Barry	11,546
10	Dirk Nowitzki	47,053	Derek Fisher	38,318	Ricky Davis	28,802	Antoine Wright	22,124	Bobby Jackson	17,040	Joel Anthony	11,468
11	Vince Carter	46,936	Marvin Williams	38,152	Al Thornton	28,749	Darko Milicic	22,106	Sasha Vujacic	16,880	Ronnie Price	11,402
12	Deron Williams	46,845	Troy Murphy	38,081	Charlie Villanueva	28,184	Darius Songaila	22,103	Carlos Arroyo	16,803	Malik Allen	11,259
13	Stephen Jackson	46,780	Rafer Alston	37,792	Kyle Korver	28,084	Zaza Pachulia	22,071	Mark Blount	16,769	Chris Quinn	11,230
14	Raymond Felton	46,622	Kevin Martin	36,967	Brendan Haywood	27,983	Spencer Hawes	21,991	Kevin Love	16,669	Dan Gadzuric	11,164
15	Steve Nash	46,241	Andrea Bargnani	36,872	Kenyon Martin	27,980	Kelenna Azubuike	21,909	Joey Graham	16,504	Ruben Patterson	11,139
16	Danny Granger	44,800	Earl Watson	36,624	Trevor Ariza	27,905	Ime Udoka	21,876	Lou Williams	16,432	Mardy Collins	11,109
17	Rashard Lewis	44,796	Steve Blake	36,609	Michael Finley	27,314	Ronny Turiaf	21,834	Tony Allen	16,380	Hilton Armstrong	11,098
18	Carmelo Anthony	44,607	Corey Maggette	36,458	Maurice Evans	27,273	Desmond Mason	21,825	J.J. Redick	16,281	Shaun Livingston	11,069
19	Richard Jefferson	44,195	Udonis Haslem	36,362	Mickael Pietrus	27,156	Jamario Moon	21,783	Matt Harpring	16,124	Brandon Jennings	11,028
20	Amare Stoudemire	44,009	Devin Harris	36,074	Erick Dampier	27,145	Devin Brown	21,695	Jannero Pargo	16,092	Greg Buckner	10,956
21	John Salmons	43,992	Richard Hamilton	35,991	Mike Conley	27,132	Marc Gasol	21,243	Johan Petro	16,085	Louis Williams	10,837
22	Caron Butler	43,968	Kevin Garnett	35,817	Tracy McGrady	27,095	Luke Walton	21,210	Daequan Cook	16,052	Tyronn Lue	10,799
23	Baron Davis	43,896	Brad Miller	35,648	Andray Blatche	27,074	Marquis Daniels	21,140	Anthony Morrow	16,044	Sam Cassell	10,783
24	Josh Smith	43,851	Tony Parker	35,389	Elton Brand	26,906	Kurt Thomas	20,910	Brandon Bass	16,016	Shannon Brown	10,719
25	David West	43,690	Chris Duhon	35,372	O.J. Mayo	26,888	Gilbert Arenas	20,868	Jerry Stackhouse	15,986	Antoine Walker	10,706
26	Shawn Marion	43,629	Jeff Green	34,935	Thaddeus Young	26,800	Eddie House	20,834	DeSagana Diop	15,962	Jonny Flynn	10,695
27	Hedo Turkoglu	43,429	Rasheed Wallace	34,399	Nate Robinson	26,773	Trenton Hassell	20,735	Stephon Marbury	15,942	Travis Diener	10,637
28	Gerald Wallace	43,407	Rasual Butler	34,381	Travis Outlaw	26,601	Eric Gordon	20,699	Dorell Wright	15,676	Damon Jones	10,551
29	Ray Allen	43,312	Jose Calderon	34,105	Sebastian Telfair	26,548	Anthony Carter	20,679	Nazr Mohammed	15,627	Yakhouba Diawara	10,507
30	Chris Bosh	43,234	Raja Bell	34,066	Damien Wilkins	26,462	Joe Smith	20,623	Earl Boykins	15,455	Louis Amundson	10,496
31	Jamal Crawford	43,109	Nick Collison	33,773	Thabo Sefolosha	26,415	Antonio Daniels	20,580	Sergio Rodriguez	15,429	Ryan Hollins	10,496
32	Chauncey Billups	43,101	Andrew Bogut	33,578	Michael Redd	26,288	Vladimir Radmanovic	20,564	Brevin Knight	15,427	Gerald Green	10,446
33	Boris Diaw	42,703	Ben Wallace	33,548	Andrew Bynum	26,127	Joel Przybilla	20,361	Jose Juan Barea	15,270	Donte Greene	10,406
34	David Lee	42,058	Beno Udrih	33,220	Shaquille O'Neal	26,077	Quinton Ross	20,277	Bobby Simmons	15,147	Brian Scalabrine	10,220
35	Jason Terry	42,027	Charlie Bell	32,791	Francisco Garcia	25,833	Jason Thompson	20,148	Luc Richard Mbah a Mo	14,977	Damon Stoudamire	10,033
36	Jason Richardson	41,972	Chris Kaman	32,745	Mikki Moore	25,590	Corey Brewer	19,934	Rashad McCants	14,961	J.J. Hickson	10,000
37	Lamar Odom	41,838	Mike Dunleavy	32,577	Keith Bogans	25,362	Rasho Nesterovic	19,823	James Jones	14,839	Will Bynum	9,989
38	Monta Ellis	41,190	Andrei Kirilenko	32,491	Roger Mason	25,261	Luther Head	19,741	George Hill	14,710	Marco Belinelli	9,987
39	Anthony Parker	41,146	Matt Barnes	32,362	Derrick Rose	25,171	Morris Peterson	19,736	Chucky Atkins	14,471	Chris Douglas-Roberts	9,683
40	Al Harrington	41,112	Leandro Barbosa	32,337	Channing Frye	25,100	Jared Dudley	19,490	Chris Andersen	14,460	Marreese Speights	9,650
41	Emeka Okafor	41,065	Paul Millsap	32,061	Jason Kapono	25,015	Nenad Krstic	19,399	Juan Dixon	14,353	Kevin Ollie	9,635
42	Tayshaun Prince	40,991	Kendrick Perkins	31,802	Ronald Murray	24,884	Yi Jianlian	19,211	Jason Collins	14,319	Nicolas Batum	9,630
43	Ben Gordon	40,867	Nene Hilario	31,676	Bruce Bowen	24,745	Carl Landry	19,159	Devean George	14,240	Julian Wright	9,560
44	Paul Pierce	40,827	J.R. Smith	31,579	Chris Wilcox	24,716	Brandon Rush	18,997	Glen Davis	14,098	Taj Gibson	9,535
45	Pau Gasol	40,827	Jermaine O'Neal	31,564	Wilson Chandler	24,707	Nick Young	18,976	Danilo Gallinari	13,895	Eddie Jones	9,527
46	Shane Battier	40,740	Jameer Nelson	31,478	Tyrus Thomas	24,604	Matt Carroll	18,958	Rudy Fernandez	13,885	Ryan Anderson	9,435
47	Jarrett Jack	40,737	Tyson Chandler	31,467	Jeff Foster	24,261	Sasha Pavlovic	18,891	Stephen Curry	13,801	Goran Dragic	9,414
48	Mo Williams	40,617	Al Horford	31,233	Rodney Stuckey	24,191	Anthony Johnson	18,781	Shelden Williams	13,731	Jonas Jerebko	9,322
49	Tim Duncan	40,522	Josh Howard	31,201	Russell Westbrook	24,190	Ramon Sessions	18,706	Brian Skinner	13,715	Darren Collison	9,252
50	Chris Paul	40,417	Manu Ginobili	31,152	Jarvis Hayes	23,956	Reggie Evans	18,599	D.J. Augustin	13,689	James Singleton	9,169
51	Luol Deng	40,259	Anderson Varejao	31,125	Cuttino Mobley	23,915	Eduardo Najera	18,416	Roy Hibbert	13,626	JaVale McGee	9,120
52	Ryan Gomes	40,214	Andris Biedrins	30,943	Chuck Hayes	23,911	Josh Boone	18,415	Amir Johnson	13,586	Eric Snow	9,088
53	Al Jefferson	40,184	Quentin Richardson	30,442	Yao Ming	23,740	Arron Afflalo	18,406	Kwame Brown	13,554	Solomon Jones	9,052
54	Marcus Camby	40,171	Hakim Warrick	30,398	Jared Jeffries	23,737	Rodney Carney	18,385	Royal Ivey	13,401	Omri Casspi	8,748
55	Dwyane Wade	40,132	Willie Green	30,335	Craig Smith	23,697	Matt Bonner	18,327	Bostjan Nachbar	13,294	Kenny Thomas	8,659
56	Brandon Roy	40,028	James Posey	30,109	Kyle Lowry	23,638	Courtney Lee	17,996	Dominic McGuire	13,117	Stromile Swift	8,634
57	Ron Artest	39,947	Randy Foye	29,993	Keyon Dooling	23,530	Mario Chalmers	17,965	Marcus Williams	13,086	Juan Carlos Navarro	8,621
58	Mehmet Okur	39,831	Drew Gooden	29,973	Martell Webster	23,281	C.J. Watson	17,950	Adam Morrison	13,011	Jacque Vaughn	8,568
59	Rajon Rondo	39,647	Larry Hughes	29,899	Joakim Noah	23,155	Juwan Howard	17,883	Francisco Elson	12,653	Bonzi Wells	8,544
60	Grant Hill	39,532	Delonte West	29,880	Tim Thomas	23,000	Fred Jones	17,789	Smush Parker	12,626	Anthony Randolph	8,543

Description of the Model

In our Skills Plus Minus ("SPM") framework, we run a series of nested probit regressions to estimate the likelihood of various events for a given play. We order a series of events $\{EVTi,\ i = 1,\ldots n\}$ sequentially. We then define Pr_{EVTi}, the conditional probability of each $EVTi$ occurring, as:

$$\mathcal{N}(\mu_{EVTi} + B_{EVTiHC}HC + \sum_{n=1}^{3} B_{EVTiSTn} \cdot STn + \sum_{n=1}^{360} B_{EVTiOFFn} \cdot OFFn + \sum_{n=1}^{360} B_{EVTiDEFn} \cdot DEFn)$$

Pr_{EVTi} is the probability of the event i, conditional on all prior events in the sequence not occurring (since only one event can occur per play). $\mathcal{N}(\cdot)$ is the cdf of the standard normal distribution, μ_{EVTi} is a constant associated with the event, HC is the home court dummy variable, STn is the possession start variable, and $OFFn$ and $DEFn$ are player dummy variables. HC is 1 if the home team has possession, and 0 if the away team has possession. STn are dummy variables for either "Defensive Rebound", "Offensive Rebound", or "Steal".

"Out of Bounds" has been normalized to 0. $OFFn$ are the dummy variables that indicate the offensive players on the court during the play, while $DEFn$ are the dummy variables that indicate the defensive players.

We have dummy variables for the 360 players[2] who have participated in the most plays in our data sample, and define all others to be "replacement level" players. B_{EVTiHC}, $B_{EVTiSTn}$, $B_{EVTiOFFn}$, and $B_{EVTiDEFn}$ are coefficients associated with the variables, for event i. Each player has two ratings in any given event: offense and defense. Table 5 displays the regression results for steals.

Table 5. Probit Estimation of Steals

$$\mathcal{N}(\mu_{STL} + B_{STLHC}HC + \sum_{n=1}^{3} B_{STLSTn} \cdot STn + \sum_{n=1}^{360} B_{STLOFFn} \cdot OFFn + \sum_{n=1}^{360} B_{STLDEFn} \cdot DEFn)$$

	Estimate	Std. Err.	z value	Pr(>|z|)
(Intercept)	-1.4230	0.0187	-76.02	0.0%
Home Court	-0.0128	0.0039	-3.29	0.1%
Dreb	0.0598	0.0045	13.39	0.0%
Oreb	-0.1336	0.0069	-19.29	0.0%
Steal	0.0130	0.0082	1.57	11.5%

Offense	Estimate	Std. Err.	z value	Pr(>|z|)	Defense	Estimate	Std. Err.	z value	Pr(>|z|)
Chris Paul	-0.1483	0.0269	-5.51	0.0%	Thabo Sefolosha	0.1169	0.0207	5.66	0.0%
Vince Carter	-0.0927	0.0191	-4.85	0.0%	Trevor Ariza	0.1045	0.0201	5.19	0.0%
Leandro Barbosa	-0.0951	0.0200	-4.76	0.0%	Renaldo Balkman	0.1348	0.0265	5.09	0.0%
Kobe Bryant	-0.1149	0.0247	-4.65	0.0%	Gerald Wallace	0.1063	0.0214	4.96	0.0%
Joe Johnson	-0.1141	0.0249	-4.59	0.0%	C.J. Watson	0.1156	0.0239	4.84	0.0%
Tyreke Evans	-0.1521	0.0343	-4.43	0.0%	Chuck Hayes	0.1051	0.0226	4.66	0.0%
Stephon Marbury	-0.1132	0.0293	-3.86	0.0%	Ronnie Brewer	0.0970	0.0209	4.63	0.0%
LeBron James	-0.0856	0.0223	-3.84	0.0%	Monta Ellis	0.0856	0.0190	4.50	0.0%
Rajon Rondo	-0.0959	0.0254	-3.78	0.0%	Devin Harris	0.0929	0.0211	4.41	0.0%
Jannero Pargo	-0.0982	0.0261	-3.77	0.0%	Thaddeus Young	0.0939	0.0215	4.36	0.0%
… best 10 above, worst 10 below …					… best 10 above, worst 10 below …				
Dwight Howard	0.0739	0.0232	3.18	0.1%	J.J. Hickson	-0.1060	0.0340	-3.11	0.2%
Bonzi Wells	0.0981	0.0304	3.22	0.1%	Fabricio Oberto	-0.0835	0.0260	-3.21	0.1%
Brook Lopez	0.0955	0.0293	3.26	0.1%	Andres Nocioni	-0.0672	0.0198	-3.39	0.1%
Shaquille O'Neal	0.0652	0.0199	3.28	0.1%	Jermaine O'Neal	-0.0739	0.0203	-3.63	0.0%
Louis Amundson	0.1078	0.0327	3.30	0.1%	Wally Szczerbiak	-0.0909	0.0245	-3.71	0.0%
Andris Biedrins	0.0712	0.0206	3.45	0.1%	Joel Anthony	-0.1145	0.0301	-3.80	0.0%
Ryan Hollins	0.0993	0.0279	3.56	0.0%	Andrea Bargnani	-0.0809	0.0207	-3.91	0.0%
Andrew Bogut	0.0866	0.0242	3.57	0.0%	Amare Stoudemire	-0.1041	0.0241	-4.31	0.0%
Chris Kaman	0.0921	0.0204	4.51	0.0%	Erick Dampier	-0.1109	0.0248	-4.48	0.0%
Eddy Curry	0.1471	0.0286	5.15	0.0%	Mike Miller	-0.0918	0.0202	-4.54	0.0%

For example, if Rajon Rondo plays on the road on a team with four other replacement level players, against a team with five replacement level players, the probability of a steal for a possession that started out-of-bounds would be:

$$Pr_{STL} = \mathcal{N}(-1.423 - 0.0128 * 0 - 0.0959) = 6.4\% \text{ if Rondo's team has the ball}$$

$$Pr_{STL} = \mathcal{N}(-1.423 - 0.0128 * 1 + 0.0937) = 9.0\% \text{ if Rondo's opponent has the ball}$$

[2] We use 360 players since there are 30 NBA teams and twelve players are allowed to play in a given game. Thus, replacement players are those who would likely be the worst player on any team. If we change the number of players, then the PORP numbers will change, since the cutoff for a replacement player will be different. The other results, including synergies calculated, however, will not be materially different.

We bucket each event into the following "skill" categories:

Ball-handling Category: Steal, Non-steal turnover

Rebounding Category: Rebound of a missed field goal, Rebound of a missed free throw

Scoring Category: Made field goal (2 or 3 points), Missed field goal, Made free throw (1, 2, 3, or 4 points), Missed free throw (0, 1, 2, or 3 points).

Features of the Model

Uses simulations to estimate both mean and variance of outcomes

The SPM framework estimates how the start-of-play state variable (defensive rebound, offensive rebound, steal or out of bounds) affects the probability of an outcome. If we start a game with an out of bounds play, we are able to simulate an entire basketball game, since we can use the estimated coefficients to estimate the probability of every possible outcome and the resultant end-of-play state variable. We can then convert these simulations into winning percentages and point differentials. To rate each player, we simulate games with the player and four "replacement-level" players on one team, and five "replacement level" players on the other team.

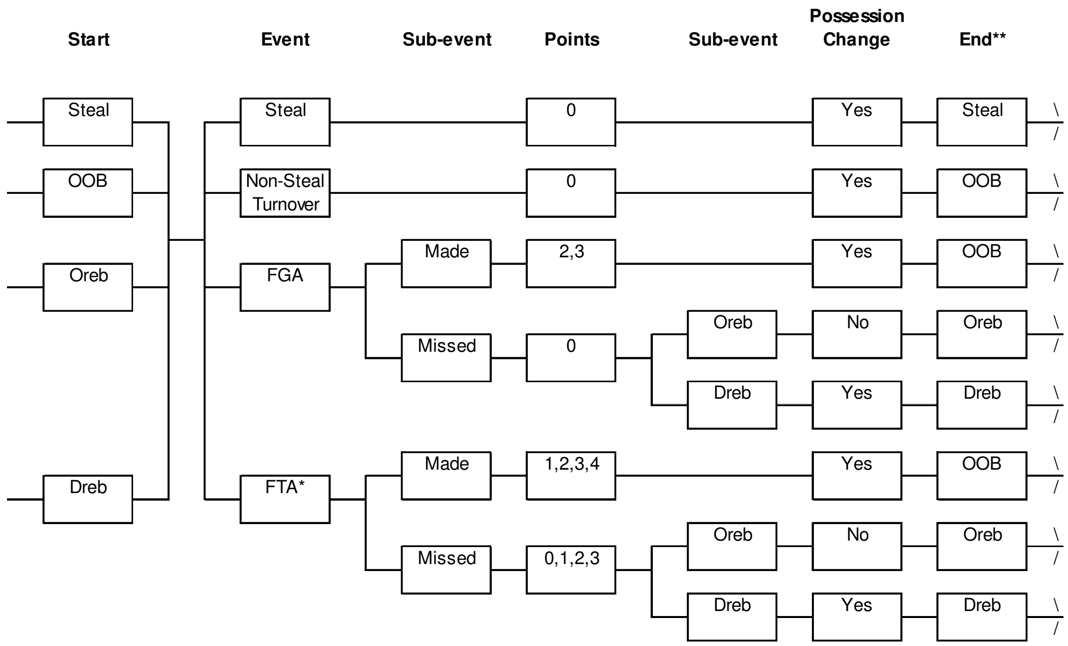

Figure 1. Flow chart of events.

Figure 1 shows the "flow chart" of the simulations. The probabilities associated with each node in the chart are calculated using the point estimates of the nested probit model we estimated. For the analysis done in this paper, we do not simulate games since each simulation is computationally time-consuming. Instead, we calculate a "steady-state" level of outcomes which would occur if a game has infinite length. We rank each player by the estimated point differential of an average length game that starts and ends in this "steady state." The results are not materially different from a simulation that starts with an out-of-bounds play. Using this

* Free throw events include "and-1" situations.

** Steals, Oreb, and Dreb sometimes end with an OOB situation if a timeout is taken or a non-shooting foul is committed, for example.

"steady state" approach, we do not calculate a range of outcomes. Instead, we calculate expected point differentials using the point estimates of the player skill parameters.

Models at the "play" level instead of the "possession level"

Imagine a situation where a team misses five consecutive field goals, and grabs five consecutive offensive rebounds, before finally making a field goal. Traditional APM will consider that sequence of events one possession which results in two points. Our SPM framework will instead count six plays, five of which end in missed field goals and offensive rebounds, and the sixth resulting in a made field goal. SPM will determine that the team with the ball has poor scoring skills but excellent offensive rebounding skills. Our framework distinguishes this sequence of events from a situation where the team immediately scores a field goal, since the outcomes were achieved in dramatically different ways. In the former scenario, the defensive team may want to counter with a defensive rebounder, while in the latter scenario, the defensive team could counter with a stronger on-the-ball defender.

Considers how a play starts

Unlike traditional APM, our framework identifies how each play starts: out-of-bounds, steal, defensive rebound or offensive rebound. We find that the start variable materially affects the outcome of the play. For example, we find that if a play starts with a steal, the average points scored increases from 0.83 to 1.04.

Reveals the strengths and weaknesses of each player

SPM provides granularity to a player's offensive and defensive ratings. If a player is a strong defender, is it because they create steals, prevent scoring, or grab defensive rebounds?

Results and Discussion

Individual Player Ratings

In this section we provide the results of the skill ratings of the 360 players who participated in the most plays in our data sample. See the Appendix for the various tables of player ratings. To estimate the contribution of each skill (e.g. steals), we isolate a player's "steals" ratings, and set his other skills to replacement levels. For example, we create a fictional player who has Ronnie Brewer's "steals" ratings, but is replacement level in all other skills. We then simulate games where one team consists of the fictional player and four replacement players, and their opponent utilizes five replacement players. The estimated point differential of this game is the player's ratings for that particular skill. For example, we estimate that Ronnie Brewer's defensive ball-handling skills are worth 3.2 points per game.

We rank the players by Points Over Replacement Player ("PORP"), the average expected point differential if the player plays an entire game with replacement players. For instance, a team with LeBron James and four replacement players would outscore a team with five replacement players by 15.1 points per game on average. The weighted average PORP across our data set is 2.82 points. The high rating of LeBron James provides some validation of our model, since many experts considered him the best player in the NBA during the four seasons in our data set[5]. Also, not surprisingly, a point guard (Chris Paul) is rated the best ball-handler, while the

[5] LeBron James received the most total votes for Most Valuable Player from 2006-2007 to 2009-2010. Source: www.basketball-reference.com.

best rebounders are generally power forwards and centers (e.g Jason Collins).

SPM Can Predict Which Skills Go Well with Each Other

To investigate synergies, we took the best players in the six skills and isolated their skills by setting their other skills to zero, or replacement level. We then tested $6 * 7/2 = 21$ combinations to see which skills have synergies. The six players are shown in Table 6.

Table 6. The best players in each of the six skills.

	Offensive	**Defensive**
Ballhandling	Chris Paul	Ronnie Brewer
Rebounding	Reggie Evans	Jason Collins
Scoring	Steve Nash	Kevin Garnett

We measured synergies by how many additional points a combination of two skills create. For example, Chris Paul's offensive ballhandling is worth 4.8 points, while Reggie Evans' offensive rebounding is worth 3.1 points. We calculate that a team with Chris Paul's offensive ballhandling and Reggie Evans' defensive rebounding will have a 8.1 point advantage. Therefore we calculate synergies as worth 0.2 points (8.1-4.8-3.1). Synergies are the difference between the point differential of the combined team and the sum of the two individual players; they tell us which types of players work well with one another. Table 7 has the results. We highlight a few of the bigger numbers.

Table 7. Synergies between skills.

	Oballhandling	Dballhandling	Oreb	Dreb	Oscore	Dscore
Oballhandling	-0.825					
Dballhandling	0.000	0.307				
Oreb	0.224	-0.052	0.293			
Dreb	0.071	-0.134	-0.002	-0.394		
Oscore	0.550	0.042	-0.191	0.254	-0.826	
Dscore	-0.064	-0.172	-0.132	0.128	-0.031	-0.284

Offensive ballhandling (preventing turnovers) has negative synergies with itself (-0.825) because a lineup with one great ballhandler does not need another. Defensive ballhandling (creating turnovers) has positive synergies with itself (0.307) because defenders who create turnovers feed off each other, creating more turnovers than they would individually. Offensive scoring has negative synergies with itself (-0.826) because players must share one ball. Defensive scoring has negative synergies with itself (-0.284) because most defensive stands end with a stop anyway.

Offensive rebounding has positive self-synergies (0.293), while defensive rebounding has negative self-synergies (-0.394). This differential sign illustrates a larger aspect of SPM. Because synergy is the excess to the total beyond the sum of the individual parts, any skill that adds to an event that is already likely to happen (such as securing a defensive rebound) will not give as much benefit as a skill that adds to an event that is unlikely to happen (such as securing an offensive rebound).

The cross-terms are more complex. Offensive ballhandling has positive synergies with offensive rebounding (0.550) because offensive ballhandling helps a team convert possessions into shot attempts, and offensive rebounding increases the number of possessions over which the ballhandler can protect the ball. Similarly, offensive ballhandling has positive synergies with offensive scoring (0.550) because the team receives more scoring opportunities, and those opportunities are good ones.

Offensive scoring has positive synergies with defensive rebounding (0.254) and negative synergies with offensive rebounding (-0.191) because defensive rebounding increases the number of potential scoring opportunities while offensive rebounding is more valuable when offensive scoring is low, since poor offensive players generate more offensive rebounding opportunities.

Empirical Evidence Suggests that Synergies Exist

Our framework predicts that skills that affect rare events (e.g. steals, offensive rebounds) will have positive synergies, while skills that contribute to common events (e.g. defensive rebounds) will have negative synergies. This feature is a result of our nested probit specification. Is this specification realistic? Do two players with strong defensive ballhandling skills create more turnovers than one? In this section, we investigate empirical evidence to validate our model.

We sorted the 987,343 observations into one hundred buckets, ordered by predicted steals. Within each bucket (each with 9873 or 9874 observations), we calculated the total predicted steals and the total actual steals. In the following scatterplot, we graph the one hundred data points, each representing a bucket of actual steals and predicted steals. If positive synergies in steals do not exist, then we would see that actual steals are less than predicted steals, for both low and high levels of predicted steals. For medium levels of predicted steals, we would see actual steals are higher than predicted steals. Instead, we see that actual steals are well within the 95% confidence intervals of predicted steals across all levels: only three points out of one hundred fall outside, two below and one above. This evidence suggests that our choice of probit to model the synergies in steals is a reasonable one.

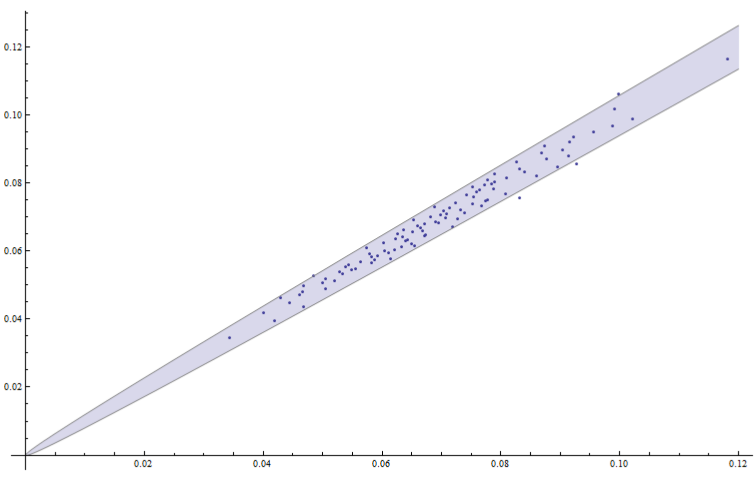

Figure 3: Actual Steals (y-axis) versus Predicted Steals (x-axis), with 95% probability confidence bands

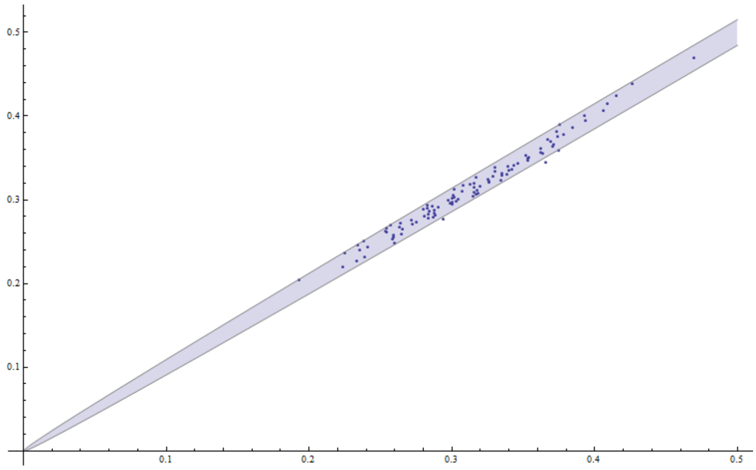

Figure 4: Actual Offensive Rebounds (y-axis) versus Predicted Offensive Rebounds (x-axis), with 95% probability confidence bands

Our framework also predicts that offensive rebounding has positive synergies with itself. Using the same methodology, we plot actual offensive rebounds versus predicted offensive rebounds. We have 407,154 missed field goals in our data set, so that each bucket contains 4,071 or 4072 observations. The above scatterplot shows that only four points out of one hundred fall outside the 95% confidence bands. These two scatterplots suggest that positive synergies do exist for both steals and offensive rebounds, as our framework predicts.

SPM Can Be Used to Calculate Synergies for Each NBA Team

For each NBA team, we formed lineups using the top five players in terms of plays played in our data sample. We calculated their ratings individually and as the 5-player lineup. For a given lineup of players x1, x2, x3, x4 and x5, define PORP(x1,x2,x3,x4,x5) to be the estimated point differential between a game played by this team of players against a lineup of replacement players ("RP").

We then define synergies as the difference of the sum-of-the-parts from the team total:

$$\begin{aligned} &PORP(x1, x2, x3, x4, x5) - PORP(x1, RP, RP, RP, RP) - PORP(x2, RP, RP, RP, RP) \\ &\quad - PORP(x3, RP, RP, RP, RP) - PORP(x4, RP, RP, RP, RP) \\ &\quad - PORP(x5, RP, RP, RP, RP) \end{aligned}$$

The results are in Table 8. Orlando's lineup has the highest amount of synergies, over one point per game, while Minnesota's negative synergies cost their lineup just under one point per game. Using the Pythagorean expectation formula with coefficients between 14 and 16.5 (c.f. Morey 1993), 1-2 points per game can translate into 3-6 wins per season (for a team that would otherwise score and allow 100 points per game). Thus a team that consistently fields a highly positively synergistic lineup will win up to six games more than if it consistently fields a highly negatively synergistic lineup. Such a differential could be the difference between making or missing the playoffs.

To investigate why Orlando's lineup has positive synergies, we replace players from their lineup one-by-one with replacement players and see how the synergies change. We find that Jameer Nelson and Hedo Turkoglu play well together. Our framework suggests that Nelson's superior ballhandling skills complement Turkoglu's offensive skills, since Nelson gives

Turkoglu more chances to score.

Using the same method, we find that Minnesota's Ryan Gomes and Randy Foye are not good fits since they are both good offensive players who protect the ball well. As noted earlier, our framework predicts negative synergies for both offense (since the players must share the ball) and offensive ball-handling (since one good ball-handler is enough for one lineup).

Table 8. Synergies within teams.

	Player1	Player2	Player3	Player4	Player5	Separate	Combined	Synergies
ORL	D. Howard	R. Lewis	H. Turkoglu	J. Nelson	K. Bogans	24.3	25.6	1.2
CLE	L. James	A. Varejao	Z. Ilgauskas	D. Gibson	M. Williams	30.7	31.8	1.1
IND	D. Granger	T. Murphy	M. Dunleavy	J. Foster	B. Rush	18.2	19.3	1.1
DEN	C. Anthony	N. Hilario	J. Smith	K. Martin	A. Iverson	14.9	16.0	1.1
SAC	K. Martin	B. Udrih	J. Salmons	F. Garcia	B. Miller	12.9	14.0	1.0
NOK	D. West	C. Paul	P. Stojakovic	T. Chandler	R. Butler	23.0	23.8	0.8
DAL	D. Nowitzki	J. Terry	J. Howard	J. Kidd	E. Dampier	25.4	26.0	0.6
LAL	K. Bryant	L. Odom	D. Fisher	P. Gasol	A. Bynum	28.1	28.6	0.4
NJN	V. Carter	D. Harris	B. Lopez	R. Jefferson	J. Kidd	23.6	24.0	0.4
SEA	K. Durant	J. Green	N. Collison	E. Watson	R. Westbrook	18.6	18.8	0.2
DET	T. Prince	R. Hamilton	R. Wallace	R. Stuckey	J. Maxiell	15.4	15.5	0.1
BOS	P. Pierce	R. Rondo	R. Allen	K. Perkins	K. Garnett	29.4	29.5	0.0
UTA	D. Williams	M. Okur	C. Boozer	A. Kirilenko	P. Millsap	25.0	25.0	0.0
HOU	S. Battier	L. Scola	R. Alston	T. McGrady	C. Hayes	22.3	22.3	0.0
GSW	M. Ellis	A. Biedrins	S. Jackson	B. Davis	K. Azubuike	18.0	18.0	0.0
PHI	A. Iguodala	S. Dalembert	A. Miller	W. Green	T. Young	18.6	18.5	-0.1
CHA	R. Felton	G. Wallace	E. Okafor	B. Diaw	M. Carroll	13.2	13.1	-0.2
LAC	C. Kaman	A. Thornton	C. Mobley	E. Gordon	B. Davis	10.2	10.0	-0.2
TOR	C. Bosh	A. Bargnani	J. Calderon	A. Parker	R. Nesterovic	19.1	18.9	-0.2
CHI	L. Deng	K. Hinrich	B. Gordon	D. Rose	J. Noah	19.8	19.5	-0.3
MIA	D. Wade	U. Haslem	M. Chalmers	M. Beasley	D. Cook	18.0	17.7	-0.4
NYK	D. Lee	N. Robinson	W. Chandler	J. Crawford	J. Jeffries	14.3	13.9	-0.4
ATL	J. Johnson	J. Smith	M. Williams	A. Horford	M. Bibby	20.0	19.6	-0.4
PHX	S. Nash	A. Stoudemire	L. Barbosa	G. Hill	R. Bell	26.2	25.6	-0.6
POR	B. Roy	L. Aldridge	T. Outlaw	S. Blake	M. Webster	19.6	19.0	-0.6
MEM	R. Gay	M. Conley	O. Mayo	H. Warrick	M. Gasol	10.0	9.4	-0.6
WAS	A. Jamison	C. Butler	D. Stevenson	A. Blatche	B. Haywood	18.2	17.6	-0.6
MIL	A. Bogut	C. Bell	M. Redd	C. Villanueva	M. Williams	14.6	13.9	-0.7
SAS	T. Duncan	T. Parker	M. Ginobili	M. Finley	B. Bowen	25.8	25.1	-0.7
MIN	R. Gomes	A. Jefferson	R. Foye	C. Brewer	C. Smith	8.2	7.3	-0.8

SPM Gives Context Dependent Player Ratings

An implication of the SPM framework is that player values depend upon the other players on the court. To illustrate this concept, we took the top four players in terms of plays played for each team. We then put everyone else into a "free agent" pool. For each team, we calculated which free agent would be the best fit for the remaining four players. In this analysis, Kevin Garnett is a "free agent" because he switched teams from Minnesota to Boston in our data sample, and played only the fifth highest number of minutes for Boston. Not surprisingly, he would be the most coveted free agent by every single team. Russell Westbrook, a "free agent" because he played only two seasons in our data sample, is likewise highly coveted. There are, however, significant differences among the more marginal players. For example, Eddie Jones, although retired, would fit well in a team like Minnesota (who rank him the fourth most desirable free agent), but would not fit in on the Spurs (who rank him seventeenth). Likewise, Marcus Camby would be coveted by the Knicks or Nets (ranked sixth), but not by the Pacers (ranked nineteenth).

Table 9 shows the "free agent" fits for each team.

Table 9: "Free agents" and synergies.

	Top Choice	2nd Choice	3rd Choice	4th Choice	5th Choice	6th Choice
CHI	K. Garnett	R. Westbrook	A. Johnson	N. Batum	R. Hibbert	C. Billups
PHX	K. Garnett	R. Hibbert	A. Johnson	R. Westbrook	N. Batum	B. Jennings
ATL	K. Garnett	A. Johnson	R. Westbrook	R. Hibbert	N. Batum	E. Jones
HOU	K. Garnett	A. Johnson	R. Westbrook	R. Hibbert	N. Batum	T. Young
IND	K. Garnett	R. Westbrook	A. Johnson	N. Batum	C. Billups	B. Jennings
LAC	K. Garnett	A. Johnson	R. Westbrook	N. Batum	R. Hibbert	C. Billups
MIL	K. Garnett	A. Johnson	R. Westbrook	R. Hibbert	N. Batum	T. Young
NOK	K. Garnett	A. Johnson	R. Westbrook	N. Batum	R. Hibbert	C. Billups
NYK	K. Garnett	R. Westbrook	A. Johnson	R. Hibbert	N. Batum	M. Camby
POR	K. Garnett	R. Westbrook	A. Johnson	N. Batum	R. Hibbert	C. Billups
TOR	K. Garnett	R. Westbrook	A. Johnson	R. Hibbert	N. Batum	C. Billups
WAS	K. Garnett	R. Westbrook	A. Johnson	R. Hibbert	N. Batum	C. Billups
DEN	K. Garnett	R. Westbrook	C. Billups	N. Batum	A. Johnson	B. Jennings
SAS	K. Garnett	A. Johnson	R. Westbrook	R. Hibbert	N. Batum	Y. Ming
CHA	K. Garnett	A. Johnson	R. Westbrook	N. Batum	R. Hibbert	T. Young
CLE	K. Garnett	R. Westbrook	A. Johnson	N. Batum	R. Hibbert	C. Billups
DET	K. Garnett	A. Johnson	R. Westbrook	R. Hibbert	N. Batum	T. Young
MIN	K. Garnett	A. Johnson	R. Westbrook	E. Jones	N. Batum	R. Hibbert
NJN	K. Garnett	R. Westbrook	A. Johnson	N. Batum	R. Hibbert	M. Camby
PHI	K. Garnett	A. Johnson	R. Westbrook	R. Hibbert	N. Batum	T. Young
SAC	K. Garnett	R. Westbrook	N. Batum	C. Billups	A. Johnson	R. Hibbert
SEA	K. Garnett	R. Westbrook	A. Johnson	R. Hibbert	C. Billups	N. Batum
UTA	K. Garnett	R. Westbrook	A. Johnson	R. Hibbert	N. Batum	T. Young
BOS	K. Garnett	R. Westbrook	A. Johnson	N. Batum	R. Hibbert	T. Young
DAL	K. Garnett	R. Westbrook	N. Batum	A. Johnson	R. Hibbert	C. Billups
MEM	K. Garnett	R. Westbrook	A. Johnson	C. Billups	N. Batum	E. Jones
LAL	K. Garnett	R. Westbrook	A. Johnson	R. Hibbert	N. Batum	C. Billups
MIA	K. Garnett	R. Westbrook	A. Johnson	N. Batum	R. Hibbert	E. Jones
ORL	K. Garnett	R. Westbrook	A. Johnson	N. Batum	R. Hibbert	B. Jennings
GSW	K. Garnett	R. Westbrook	A. Johnson	N. Batum	R. Hibbert	T. Young

Using SPM to Find Mutually Beneficial Trades

Other player rating systems like WP or Win Shares (see Oliver 2004) cannot generate ex-ante mutually beneficial trades because one player is always ranked higher than another (unless the distribution of minutes is changed). In contrast, the SPM framework can generate mutually beneficial trades because each potential lineup has different synergies. We examined every possible two player trade from one team's starting five to another team's starting five. There are a total of $30 \cdot 29/2 = 435$ possible team trading partners. Each pair of teams has $5 \cdot 5 = 25$ possible trades, so there are $435 * 25 = 10{,}875$ possible trades. We found 222 mutually beneficial trades, or 2% of all possible trades. These trades do not consider the distribution of minutes or the composition of the team's bench. Table 10 lists a few trades.

Figure 2 shows the network of the 222 mutually beneficial trades among the various teams. Not surprisingly, the teams with the lowest synergies (Minnesota and San Antonio) have the most possible trading partners and are near the interior of this "trade network". Meanwhile the teams with the highest synergies (Orlando and Cleveland) have the fewest trading partners and are on the perimeter.

Why is Chris Paul for Deron Williams a mutually beneficial trade? Overall, our SPM ratings rate Chris Paul and Deron Williams nearly the same, but with differences in skills. Paul is a better ballhandler, Williams a slightly better rebounder, and Williams is better at offense and defense. See Table 11.

Table 10. Some mutually beneficial trades.

Team 1	Team 2	Player 1	Player 2
PHX	MIN	Amare Stoudemire	Ryan Gomes
PHX	DAL	Amare Stoudemire	Erick Dampier
NOK	UTA	Chris Paul	Deron Williams
MIN	BOS	Al Jefferson	Rajon Rondo
MIN	MIA	Ryan Gomes	Udonis Haslem
MIN	MIA	Corey Brewer	Daequan Cook
DET	LAL	Rodney Stuckey	Derek Fisher
HOU	MIN	Luis Scola	Ryan Gomes
MIL	MIN	Mo Williams	Al Jefferson
ATL	MIA	Marvin Williams	Daequan Cook

Table 11. Comparison of Chris Paul and Deron Williams

	Off Ballhand.	Def Ballhand.	Off Rebound.	Def Rebound.	Off Scoring	Def Scoring
Chris Paul	4.8	1.2	-0.4	-1.4	4.7	-0.9
Deron Williams	1.9	-0.3	-1.7	0.1	6.5	1.4

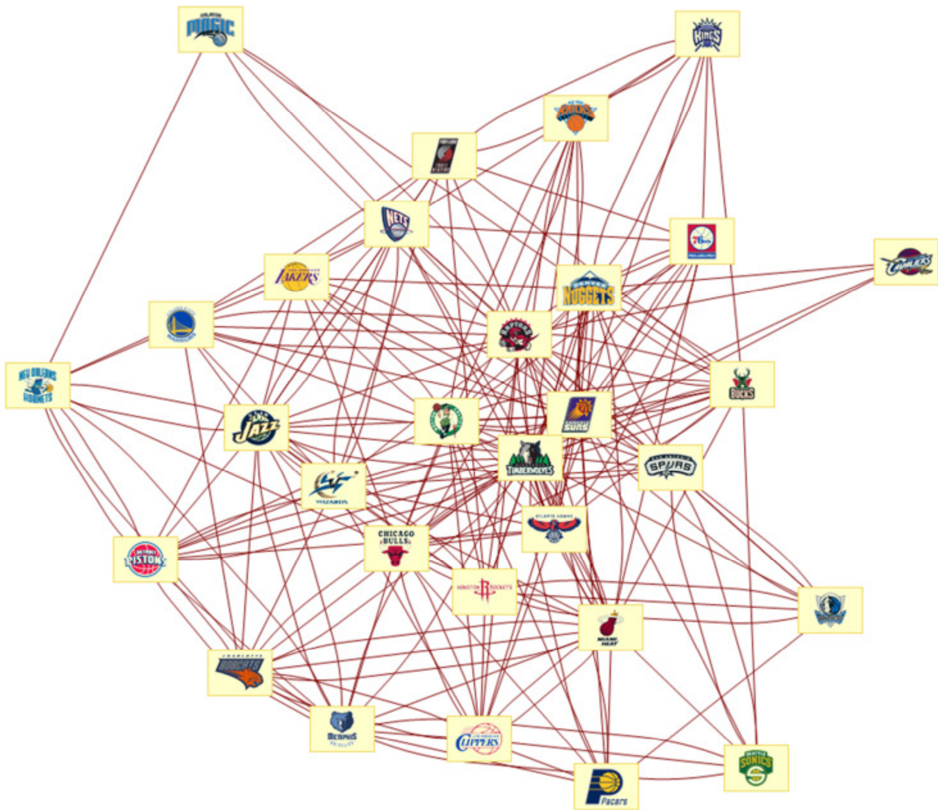

Figure 2. Trade network of mutually beneficial trades.

The SPM framework predicts that Chris Paul is a better fit for Utah because he creates a lot of steals (3.1 steals per 48 minutes ("SP48M")), while no one else in the New Orleans lineup does (West 1.0 SP48M, Stojakovic 1.1, Chandler 0.7, Butler 0.9). Utah, on the other hand, has many players who create steals (Kirilenko 2.0, Boozer 1.5, Millsap 1.7, Okur 0.9, Williams 1.4). Because defensive steals has positive synergies in our system, Chris Paul's ballhawking skills fit better in Utah, where he can team up with others and wreak havoc to opponents' ballhandlers.

Conversely, why would New Orleans trade for Deron Williams? Our framework predicts that Williams is a better offensive fit with New Orleans. There are negative synergies between two good offensive players since they must share only one ball, and the New Orleans starters take fewer shots than Utah’s. At New Orleans, Deron Williams would not need to share the ball with so many players.

The Utah lineup of Williams (PG), Okur (F-C), Boozer (F-C), Kirilenko (F) and Millsap (F) may seem big. The next player on Utah’s roster in terms of plays in our sample is Ronnie Brewer (G-F). If we substitute Millsap for Brewer, the case for a Deron Williams for Chris Paul trade becomes stronger, since Brewer is good at steals (2.7 SP48M).

Conclusion

We provide a novel Skills Plus Minus (“SPM”) framework that can be used to measure synergies within basketball lineups, provide roster-dependent rankings of free agents, and generate mutually beneficial trades. To our knowledge, the SPM framework is the first system that can generate ex-ante mutually beneficial trades without a change in the minutes played. Other ranking systems cannot generate mutually beneficial trades because one player is always ranked ahead of another.

Future research could use the SPM framework to calculate the optimal substitution patterns that maximize overall synergies given a fixed distribution of minutes played to each player, highlight the risks and exposures each team with respect to the specific skills, and evaluate the possibility of a separate synergy factor of players that may improve the skills of their teammates by even more than would be suggested by the synergies of the skills.

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Appendix: Player Ratings

Best and Worst Overall

Best	PORP	Worst	PORP
LeBron James	15.1	Johan Petro	-3.3
Steve Nash	14.3	Gerald Green	-3.3
Dwyane Wade	13.5	Joel Anthony	-3.8
Kevin Garnett	13.3	Brian Skinner	-4.5
Kobe Bryant	10.2	Dominic McGuire	-4.5
Dirk Nowitzki	9.7	Hakim Warrick	-4.9
Tim Duncan	9.6	Earl Boykins	-5.4
Chris Bosh	9.5	Eddy Curry	-6.7
Manu Ginobili	9.4	Josh Powell	-7.8
Russell Westbrook	9.4	J.J. Hickson	-8.8

Best and Worst Offensive Ballhandling (preventing steals and turnovers)

Best	PORP	Worst	PORP
Chris Paul	4.8	Mikki Moore	-2.4
Brandon Jennings	4.6	Andrew Bogut	-2.4
Kobe Bryant	4.3	Louis Amundson	-2.5
Sasha Vujacic	3.8	Hilton Armstrong	-2.7
Sam Cassell	3.6	Kwame Brown	-2.8
LeBron James	3.3	Yao Ming	-2.8
Chauncey Billups	3.2	Ryan Hollins	-3.3
Mike Conley	3.1	Kendrick Perkins	-3.4
Daequan Cook	3.1	Joel Przybilla	-3.5
Jason Terry	3.0	Eddy Curry	-6.3

Best and Worst Defensive Ballhandling (creating steals and turnovers)

Best	PORP	Worst	PORP
Ronnie Brewer	3.2	Tim Duncan	-2.0
Gerald Wallace	2.9	Michael Finley	-2.3
Thabo Sefolosha	2.9	Brook Lopez	-2.4
Devin Harris	2.9	Aaron Brooks	-2.5
Monta Ellis	2.8	Andrew Bynum	-2.5
Renaldo Balkman	2.8	Taj Gibson	-2.6
Rajon Rondo	2.7	Joel Anthony	-2.8
Luc Richard Mbah a Moute	2.7	Amare Stoudemire	-3.3
C.J. Watson	2.7	Erick Dampier	-3.6
Eddie Jones	2.7	J.J. Hickson	-4.2

Best and Worst Offensive Rebounding

Best	PORP	Worst	PORP
Reggie Evans	3.1	Chris Quinn	-1.9
Matt Harpring	3.0	Jannero Pargo	-2.0
Kevin Love	2.9	Donte Greene	-2.0
Jeff Foster	2.7	Brandon Rush	-2.1
Jason Maxiell	2.6	Rashard Lewis	-2.3
Louis Amundson	2.5	Damon Stoudamire	-2.3
Leon Powe	2.2	Danilo Gallinari	-2.4
Amir Johnson	2.1	Travis Diener	-2.5
Joakim Noah	2.0	Stephen Curry	-2.8
Jared Jeffries	2.0	Jonny Flynn	-2.8

Best and Worst Defensive Rebounding

Best	PORP	Worst	PORP
Jason Collins	3.0	Francisco Garcia	-1.5
Tim Duncan	2.6	Sasha Vujacic	-1.5
Joel Przybilla	2.5	Eddie House	-1.6
Jeff Foster	2.5	Josh Childress	-1.6
Andrew Bogut	2.3	Dominic McGuire	-1.6
Zydrunas Ilgauskas	2.3	Darren Collison	-1.6
Nene Hilario	2.2	Charlie Bell	-1.7
Roy Hibbert	2.2	Jamaal Tinsley	-1.8
Rasho Nesterovic	2.2	Travis Diener	-2.1
Samuel Dalembert	2.0	Earl Boykins	-2.1

Best and Worst Offense (assuming no turnovers)

Best	PORP	Worst	PORP
Steve Nash	12.7	James Singleton	-2.3
Dwyane Wade	9.4	Josh Powell	-2.3
LeBron James	7.8	Hilton Armstrong	-2.4
Deron Williams	6.5	Louis Amundson	-2.4
Kevin Martin	6.4	Brian Skinner	-2.4
Kobe Bryant	6.3	Ben Wallace	-2.5
Goran Dragic	6.2	Jason Collins	-2.7
Dirk Nowitzki	5.9	Eric Snow	-3.0
Manu Ginobili	5.9	Renaldo Balkman	-3.4
Danny Granger	5.9	Nene Hilario	-3.7

Best and Worst Defense (assuming no turnovers)

Best	PORP	Worst	PORP
Kevin Garnett	6.2	Damien Wilkins	-3.0
Brendan Haywood	5.7	Josh Powell	-3.0
Tim Duncan	5.4	Kevin Martin	-3.0
Joel Przybilla	5.2	Gerald Green	-3.0
Amir Johnson	5.0	Marreese Speights	-3.2
Andrew Bogut	4.8	Juan Carlos Navarro	-3.2
Chris Andersen	4.5	Royal Ivey	-3.4
Jacque Vaughn	3.9	Jose Calderon	-3.4
Yao Ming	3.9	Sasha Vujacic	-3.7
Kendrick Perkins	3.9	Will Bynum	-4.2